

Performance of the Time-Delay Digital Tanlock Loop as PM Demodulator in Gaussian Noise

Zahir M. Hussain, *Senior Member, IEEE*

School of Electrical and Computer Engineering, RMIT, Melbourne, Victoria 3000, Australia

Email: zmhussain@ieee.org

Abstract—In a previous work we proposed a phase-lock structure called the time-delay digital tanlock loop (TDTL). This digital phase-locked loop (DPLL) performs nonuniform sampling and utilizes a constant time-delay unit instead of the constant 90-degrees phase-shifter used in conventional tanlock structures. The TDTL reduces the complexity of implementation and avoids many of the practical problems associated with the analog or digital Hilbert transformer, while it preserves the most important features of the conventional DTL (CDTL). In this paper we investigate the performance of the TDTL in demodulating phase- and frequency-modulated signals in the presence of additive Gaussian noise.

I. INTRODUCTION

Analog and digital phase-locked loops (PLL's) play a significant role in signal processing and communication systems. The PLL has a variety of applications that include filtering, frequency synthesis, frequency modulation, demodulation, signal detection, motor-speed control and many other applications [1]. Digital phase-locked loops (DPLL's) has gained a significant attention due to the obvious benefits of digital systems over their analog counterparts. In addition, DPLL's have introduced many features that analog PLL's couldn't handle. Nonuniform sampling digital phase-locked loops are the most important DPLL's as they are simple to implement and easy to model, also their analysis opens the way towards understanding DPLL's in general [2]. The digital tanlock loop (DTL), proposed in [3], has introduced several significant advantages over other nonuniform sampling DPLL's, including wider locking range and reduced sensitivity of the locking conditions to the variation of the input signal power [3]. DTL proved to be efficient for many applications in digital communications. Several modified versions of CDTL have been proposed. These versions resulted in much more efficient DPLL's with much wider locking ranges (see, for example, [4]-[6]).

In [7] we proposed a new DPLL that utilizes the DTL structure with a constant time-delay unit (to produce a phase-shifted version of the incoming signal) instead of the complicated Hilbert transformer (HT) utilized by the conventional DTL. This technique reduces the complexity of the phase-shifter and avoids the limitations and other problems that accompany the 90° phase-shifter. The new loop was called the time-delay digital tanlock loop (TDTL). Except for the linearity of the characteristic function of the phase error detector, the main advantages of CDTL are maintained by TDTL despite its reduced structure. An application of the

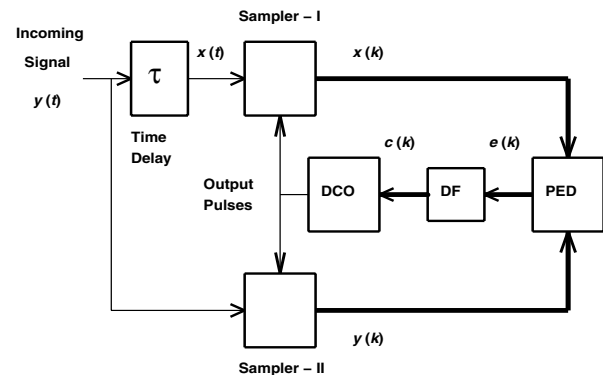


Fig. 1. A block diagram of the time-delay digital tanlock loop (TDTL). Bold lines represent multi-bit connections.

TDTL for demodulation of FSK signals was also proposed in [7].

In this paper we investigate the capability of the TDTL in demodulating angle-modulated signals in the presence of additive gaussian noise.

II. OPERATION OF THE TDTL

The TDTL is composed of a time-delay unit (τ), two samplers, a phase error detector (PED), a digital loop filter (DF), and a digital controlled oscillator (DCO) [see Fig. (1)]. At every sampling instant k , sampler I takes a sample $x(k)$ of the time-delayed version $x(t)$ of the incoming signal $y(t)$, and sampler II takes a sample $y(k)$ of the incoming signal $y(t)$. The phase error detector (PED) performs the inverse-tangent operation $\tan^{-1}[x(k)/y(k)]$ at every sampling instant k . The output of the phase error detector, $e(k)$, is a function of the phase error between the incoming signal and the digital clock at the k^{th} sampling instant in modulo 2π sense. The digital filter (DF) modifies the output of the phase error detector $e(k)$ and provides a control signal to the digital clock (digital-controlled oscillator, DCO) to decide the next sampling instant at the two samplers. As the sampling period is input-signal-dependent, this sampling process is nonuniform, and the loop arranges its frequency at the digital clock to be, in the limit, equal to the input frequency with a minimum phase difference (actually, zero phase difference for the second-order loop).

First we assume noise-free conditions. The loop input is

assumed to be a sinusoidal input signal $y(t)$ having a radian frequency ω with a frequency offset $\Delta\omega = \omega - \omega_o$ from the nominal radian frequency ω_o of the digital controlled oscillator. The input signal is expressed as follows:

$$y(t) = A \sin[\omega_o t + \theta(t)] \quad (1)$$

where A is the signal amplitude and $\theta(t) = \Delta\omega t + \theta_o$ is the phase process of the incoming signal, θ_o being a constant.

The time-delay unit causes a constant time-delay τ for the input signal. This will cause a phase lag $\psi = \omega\tau$ proportional to input radian frequency ω . The time-delayed version of the input signal, denoted by $x(t)$, will be as follows:

$$x(t) = A \sin[\omega_o t + \theta(t) - \psi] \quad (2)$$

At the k^{th} sampling instant, the sampled values of $y(t)$ and $x(t)$ are given by

$$y(k) = A \sin[\omega_o t(k) + \theta(k)] \quad (3)$$

and

$$x(k) = A \sin[\omega_o t(k) + \theta(k) - \psi] \quad (4)$$

respectively, where $\theta(k) = \theta[t(k)]$.

The sampling interval between the sampling instants $t(k)$ and $t(k-1)$ is controlled by the output of the digital filter as follows:

$$T(k) = T_o - c(k-1) \quad (5)$$

where $T_o = 2\pi/\omega_o$ is the nominal period of the digital clock and $c(i)$ is the output of the digital filter at the i^{th} sampling instant. Assuming zero initial time $t(0) = 0$, the total time $t(k)$ up to the k^{th} sampling instant is

$$t(k) = \sum_{i=1}^k T(i) = kT_o - \sum_{i=0}^{k-1} c(i) \quad (6)$$

Thus, $y(k)$ and $x(k)$ can be written as

$$y(k) = A \sin[\theta(k) - \omega_o \sum_{i=0}^{k-1} c(i)] \quad (7)$$

and

$$x(k) = A \sin[\theta(k) - \omega_o \sum_{i=0}^{k-1} c(i) - \psi] \quad (8)$$

If we define the phase error $\phi(k)$ by:

$$\phi(k) = \theta(k) - \omega_o \sum_{i=0}^{k-1} c(i) - \psi, \quad (9)$$

then we can write $y(k)$ and $x(k)$ as follows:

$$y(k) = A \sin[\phi(k) + \psi] \quad (10)$$

and

$$x(k) = A \sin[\phi(k)]. \quad (11)$$

From above equations we can show that:

$$\phi(k+1) = \phi(k) - \omega c(k) + \Lambda_o \quad (12)$$

where $\Lambda_o = 2\pi[\omega - \omega_o]/\omega_o$. This is the system equation of TDTL.

To find the *Characteristic Function* of the PED, we define $f[\alpha] = -\pi + \{(\alpha + \pi) \text{ modulo } (2\pi)\}$, then the output of the PED $e(k)$ is given by

$$e(k) = f[\text{Tan}^{-1}(\frac{\sin\{\phi(k)\}}{\sin\{\phi(k) + \psi\}})] \quad (13)$$

Thus, the characteristic function $h_\psi(\phi)$ of the phase error detector is non-linear and depends on the input frequency ω and the time delay τ ; it is given by

$$h_\psi(\phi) = f[\text{Tan}^{-1}(\frac{\sin(\phi)}{\sin(\phi + \psi)})] \quad (14)$$

The function $h(\phi)$ can equivalently be expressed in terms of the ratio $W = \omega_o/\omega$ and the nominal phase shift $\psi_o = \omega_o\tau$ as follows

$$h_\psi(\phi) = f[\text{Tan}^{-1}(\frac{\sin(\phi)}{\sin(\phi + \psi_o/W)})] \quad (15)$$

The $\text{Tan}^{-1}(x/y)$ function used by the phase detector can distinguish between the four quadrants according to the signs of x and y unlike the ordinary $\tan^{-1}(\cdot)$ function. It was shown in [7] that the function $h_\psi(\phi)$ is continuous in ϕ over the interval $(-\pi, \pi)$ for all values of ψ between 0 and π .

The first-order loop utilizes a digital filter with just a positive constant gain G_1 . In this case the system equation (12) becomes:

$$\phi(k+1) = \phi(k) - K_1' h_\psi[\phi(k)] + \Lambda_o \quad (16)$$

where $K_1' = \omega G_1$. If K_1 is defined to be $\omega_o G_1$ then $K_1' = K_1/W$.

III. PM DEMODULATION USING THE FIRST-ORDER TDTL

PM signals convey the information message $m(t)$ in the phase of a sinusoidal carrier with center frequency $\omega_o = 2\pi f_o$ such that the phase is linearly proportional to the message as follows

$$x(t) = A_o \sin[\omega_o t + \Delta_p m(t) + \gamma_o] \quad (17)$$

where Δ_p is a constant called the phase sensitivity and γ_o is the initial phase. For testing purposes we will consider the message to be a single-tone signal $m(t) = A_m \sin(\omega_m t)$ such that the PM signal will be

$$x(t) = A_o \sin[\omega_o t + \beta \sin(\omega_m t) + \gamma_o] \quad (18)$$

where $\beta = A_m \Delta_p$ is the modulation index and γ_o is a constant.

Now we consider the characteristic function of the TDTL $h(\phi)$, which is a non-linear function as shown in eq(15) except in the case when $\psi_o = \pi/2$ and $W = \omega_o/\omega = 1$, which gives $h_\psi(\phi) = \phi$. If we arrange $\psi_o = \pi/2$, then for small values of the phase error $\phi(k)$ (which can be ensured in case the frequency ratio $W = \omega_o/\omega$ is inside the lock range), the TDTL characteristic function can still be approximated as

$$h_\psi(\phi) \approx \phi \text{ for small } \phi. \quad (19)$$

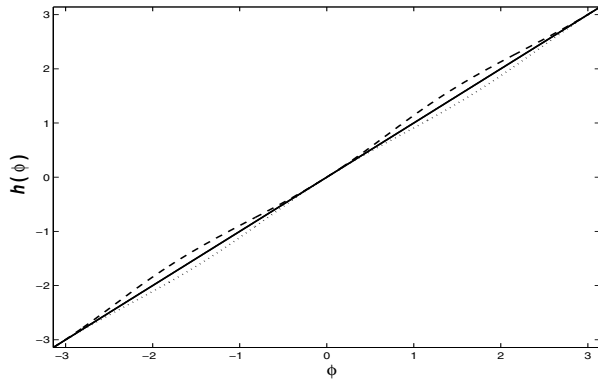


Fig. 2. Characteristic function of the time-delay digital tanlock loop (TDTL) when $\psi_o = \pi/2$. Solid line is for the frequency ratio $W = \omega_o/\omega = 1$, dashed line for $W = 1.1$, and dotted line for $W = 0.9$.

as shown in Fig. (2).

Now if we arrange the parameter K_1 in eq(16) of the first-order TDTL to be 1 and the carrier frequency to be the loop center frequency ω_o , then by using equations (9), (13), (16), (17), and (19) it can be shown that

$$m(k) \approx \frac{1}{\Delta_p} \sum_{i=0}^k e(k) + \gamma_o \quad (20)$$

Hence, a PM signal can be demodulated by adding up the PED output samples then using a low-pass analog filter for reconstruction as shown in Fig. (3). However, this is true only if the incoming frequency range is inside the locking range. In [7], the locking conditions of the first-order TDTL were given by:

$$2|1 - W| < K_1 < 2W \frac{\sin^2(\alpha) + \sin^2(\alpha + \psi_o/W)}{\sin(\psi_o/W)} \quad (21)$$

where

$$\begin{aligned} \alpha &= \tan^{-1}(\rho) \\ \rho &= \frac{\sin(\psi)\tan(\zeta)}{1 - \cos(\psi)\tan(\zeta)} = \frac{\sin(\psi)}{\cot(\zeta) - \cos(\psi)} \\ \zeta &= e_{ss} = f[\tan^{-1}(\frac{\sin(\phi_{ss})}{\sin(\phi_{ss} + \psi)})] = \frac{\Lambda_o}{K'_1} \end{aligned} \quad (22)$$

noting that e_{ss} is the steady-state output of the phase error detector and ϕ_{ss} is the steady-state value of the phase error process $\phi(k)$ as defined in eq(9).

As shown in Fig. (4), for $K_1 = 1$ the lock range can approximately be determined from the left-hand side of the above inequality.

The instantaneous frequency of the above single-tone PM signal $x(t)$ is given by

$$\omega_i(t) = d[\phi(t)]/dt = \omega_o + \beta\omega_m \cos(\omega_m t) \quad (23)$$

where the maximum and the minimum frequencies are given by $\omega_o \pm \beta\omega_m$. Using eq(21), (23) and the above discussion we

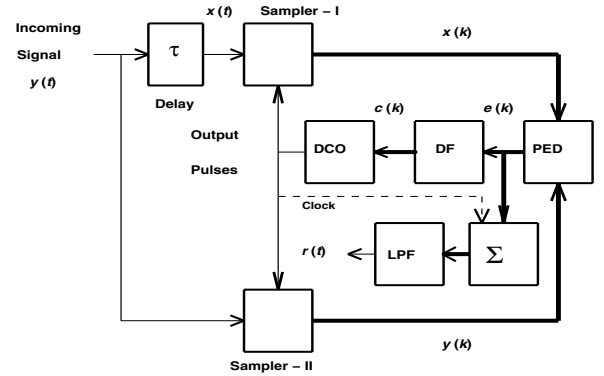


Fig. 3. Block diagram of the TDTL PM-demodulator.

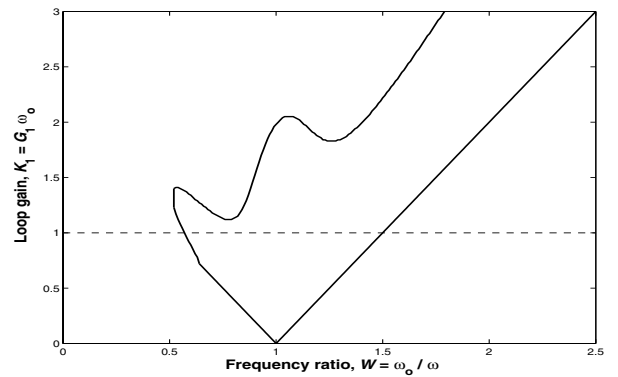


Fig. 4. Lock range of the TDTL for $\psi_o = \pi/2$.

can find the conditions for demodulating the single-tone PM as follows

$$\beta\omega_m/\omega_o < 1/3. \quad (24)$$

If a similar PM demodulation technique is implemented using the conventional DTL, a dynamic range as in eq(24) will be obtained. It is evident that, despite the reduced structure, TDTL has a performance similar to that of the DTL in many aspects. The above dynamic range in TDTL and CDTL for PM demodulation is much wider than the limit of $\beta\omega_m/\omega_o < 1/(2\pi + 1) \approx 0.13$ obtained for the sinusoidal DPLL in [9], [10].

From eq(16), the steady state phase error at the output of the PED can be given by:

$$e_{ss} = h_\psi(\phi_{ss}) = f[\tan^{-1}(\frac{\sin(\phi_{ss})}{\sin(\phi_{ss} + \psi)})] = \frac{\Lambda_o}{K'_1} \quad (25)$$

where

$$|\Lambda_o/K'_1| < \pi \quad (26)$$

from which the steady-state phase error ϕ_{ss} is given by [7]

$$\phi_{ss} = \begin{cases} \alpha & \rho \sin(\lambda) \geq 0 \\ f(\alpha + \pi) & \text{otherwise.} \end{cases} \quad (27)$$

where

$$\lambda = \Lambda_o / K_1' \quad (28)$$

$$\rho = \frac{\sin(\psi)}{\cot(\lambda) - \cos(\psi)} = \frac{\sin(\psi_o/W)}{\cot(\lambda) - \cos(\psi_o/W)} \quad (29)$$

$$\alpha = \tan^{-1}(\rho) \quad (30)$$

noting that $\tan^{-1}(\cdot)$ is the ordinary arctan function over $(-\pi/2, \pi/2)$. From eqs.(23) and (25), the PED output phase in the case of tone-PM demodulation will range between $e_{ss}|_{\max}$ and $e_{ss}|_{\min}$ as follows:

$$e_{ss}|_{\max} = \frac{\Lambda_o}{K_1'}|_{\max} = \frac{T_o}{G_1} \left[1 - \frac{1}{1 + \beta\omega_m/\omega_o} \right] \quad (31)$$

$$e_{ss}|_{\min} = \frac{\Lambda_o}{K_1'}|_{\min} = -\frac{T_o}{G_1} \left[\frac{1}{1 - \beta\omega_m/\omega_o} - 1 \right]. \quad (32)$$

The above results are generally true as long as the carrier frequency f_o is much higher than the message maximum frequency f_m such that an approximate locking can occur. This is the case in practical PM and FM transmission systems.

IV. PERFORMANCE IN GAUSSIAN NOISE

We now consider the performance of the above PM demodulator in additive Gaussian noise (AWGN) environment. In [8] we have shown that if the input signal is affected by AWGN as follows:

$$y(t) = A \sin[\omega_o t + \theta(t)] + n(t) \quad (33)$$

where $n(t)$ is additive Gaussian noise with zero mean and variance σ_n^2 , then the output of the PED, ξ , can be expressed as follows

$$\xi = e + \eta \quad (34)$$

where $e = h_\psi(\phi)$ is the deterministic output phase and η is a non-Gaussian phase noise with zero mean. The sampling index k is removed for simplicity. The pdf of the phase noise η can be given as follows:

$$\begin{aligned} \rho_{\psi,e}(\eta) &= \frac{1}{2\pi} \exp(-m_{\psi,e}\alpha) + \sqrt{\frac{m_{\psi,e}\alpha}{\pi}} \cos(\eta) \\ &\times \exp[-m_{\psi,e}\alpha \sin^2(\eta)] \\ &\times \left[\frac{1}{2} + \operatorname{erf}\{\sqrt{2m_{\psi,e}\alpha} \cos(\eta)\} \right] \end{aligned} \quad (35)$$

where

$$\alpha = A^2/2\sigma_n^2 \quad (36)$$

$$\mu_{\psi,\phi} = \frac{\sin(\psi)}{h_\psi'(\phi)} \quad (37)$$

$$m_{\psi,e} = \mu_{\psi,h_\psi^{-1}(e)} \quad (38)$$

$$\operatorname{erf}(r) = \frac{1}{\sqrt{2\pi}} \int_0^r e^{-v^2/2} dv. \quad (39)$$

When $\psi = \pi/2$, the above pdf reduces to:

$$\begin{aligned} \rho_o(\eta) &= \frac{1}{2\pi} \exp(-\alpha) + \sqrt{\frac{\alpha}{\pi}} \cos(\eta) \times \exp[-\alpha \sin^2(\eta)] \\ &\times \left[\frac{1}{2} + \operatorname{erf}\{\sqrt{2\alpha} \cos(\eta)\} \right]. \end{aligned} \quad (40)$$

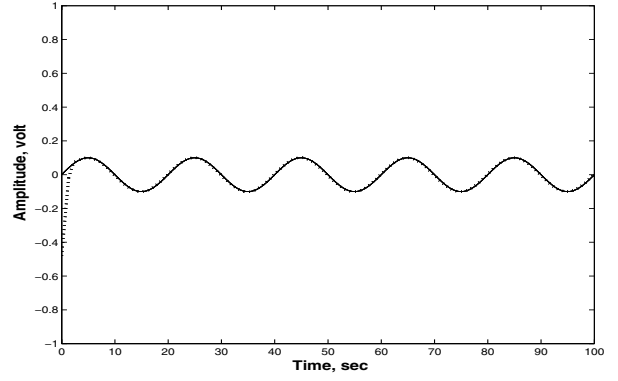


Fig. 5. Demodulation of a tone-modulated PM signal using the first-order TDTL with $A_m = 1$, $A_o = 1$, $f_o = 1$ Hz, $f_m = 0.05$ Hz, $\beta = 0.1$, $\gamma_o = -0.5$, and $\psi_o = \pi/2$.

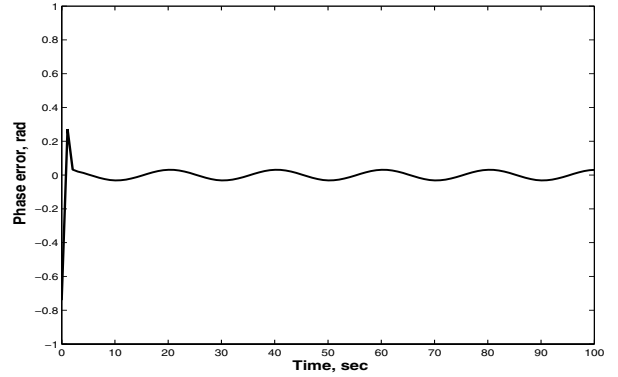


Fig. 6. Phase error process associated with the demodulation of a tone-modulated PM signal using the first-order TDTL with $A_m = 1$, $A_o = 1$, $f_o = 1$ Hz, $f_m = 0.05$ Hz, $\beta = 0.1$, $\gamma_o = -0.5$, and $\psi_o = \pi/2$.

The variance of ξ is ψ -dependent. In noisy PM demodulation, eq(20) becomes:

$$\begin{aligned} m_o(k) &\approx \frac{1}{\Delta_p} \sum_{i=0}^k \xi(k) + \gamma_o \\ &= m(k) + \frac{1}{\Delta_p} \sum_{i=0}^k \eta(k) \\ &= m(k) + n_o(k) \end{aligned} \quad (41)$$

where

$$n_o(k) = \frac{1}{\Delta_p} \sum_{i=0}^k \eta(k). \quad (42)$$

The output noise $n_o(k)$ is the sum of several non-Gaussian random variables with zero mean and different values of variance (ψ -dependent). The *Central Limit Theorem* [11] is applicable here, which confirms that $n_o(k)$ is *Gaussian* with zero mean.

V. SIMULATION RESULTS

The above system for PM demodulation has been simulated for the following incoming signal:

$$x(t) = A_o \sin[\omega_o t + \beta \sin(\omega_m t) + \gamma_o] \quad (43)$$

with $A_m = 1$, $A_o = 1$, $f_o = 1$ Hz, $f_m = 0.05$ Hz, $\beta = 0.1$, $\gamma_o = -0.5$, and $\psi_o = \pi/2$.

Except for a transient disturbance in case of a non-zero initial phase γ_o , the TDTL can efficiently demodulate the information from the carrier as shown in Fig. (5) and Fig. (6). The PED output phase error ranges between $e_{ss}|_{\max} = 0.031$ and $e_{ss}|_{\min} = -0.031$ as per eqs.(31) and (32).

Fig. (7) shows the time-domain performance of the above system in the presence of AWGN for SNR = 20 dB. Fig. (8) shows the performance of the above system in the presence of AWGN for different values of the input (received) signal-to-noise ratios (SNR's) and the modulation index β . The performance is measured in terms of the output SNR (SNR_o) versus the input SNR relation as compared to the case of baseband transmission where the two SNR's are equal. Fig. (9) shows the performance of the above system versus analog PM demodulation in which the output SNR is related to the input SNR (or baseband SNR, SNR_b) as follows [12]:

$$\text{SNR}_o = \beta^2 P_m \text{SNR}_b. \quad (44)$$

where $P_m = p_m/M^2$ is the message power p_m normalized with respect to $M^2 = [\max(m(t))]^2$, given by 1/2 for a tone PM. It is evident that the TDTL-based PM demodulation outperforms analog demodulation techniques by nearly 20 dB.

VI. CONCLUSIONS

This paper has shown that the recently proposed time-delay digital tanlock loop (TDTL) is capable of demodulating PM (and hence FM) signals in additive Gaussian noise. The basic idea was approximating the non-linear characteristic function of the loop phase error detector (and hence the system equation) under a specific arrangement of the nominal phase-shift to be a right angle. It is shown that the performance of the TDTL as a PM demodulator is comparable to that of the conventional digital tan lock loop (DTL) and is nearly 20-dB better in performance than analog PM demodulators.

REFERENCES

- [1] G. Hsieh and C. Hung, "Phase-locked loop techniques- A survey," *IEEE Trans. Ind. Elect.*, vol. 43, no. 6, pp. 609-615, Dec. 1996.
- [2] W. C. Lindsey and C. M. Chie, "A survey of digital phase-locked loops," *Proc. IEEE*, vol. 69, no.4, pp. 410-431, Apr. 1981.
- [3] J. C. Lee and C. K. Un, "Performance analysis of digital tanlock loop," *IEEE Trans. Com.*, vol. COM-30, no. 10, pp. 2398-2411, Oct.1982.
- [4] J. A. Bisson and R. W. Donaldson, "Characterization and simulation of a multisampling digital tanlock loop," *IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, pp. 354-8, 1989.
- [5] M. Hagiwara and M. Nakagawa, "Performance of the multisampling digital tanlock loop and its extension," *Electronics and Communications in Japan*, vol. 72, no. 9, pp. 45-52, 1989.
- [6] R. Tervo and R. Enriquez, "Analysis of digital tanlock loop with adaptive filtering," *IEEE Pacific Rim conference on Communications, Computers and Signal Processing*, vol. 1, pp. 5-8, 1993.

- [7] Z. M. Hussain et. al., "A time-delay digital tanlock loop," *IEEE Transactions on Signal Processing*, vol. 49, no. 8, pp. 1808-1815, Aug. 2001.
- [8] Z. M. Hussain and B. Boashash, "The time-delay digital tanlock loop: performance analysis in additive Gaussian noise," *Journal of The Franklin Institute*, vol. 399, no. 1, 2002.
- [9] H. C. Osborne, "Stability analysis of an N-th power digital phase-locked loop -Part I: First-order DPLL," *IEEE Trans. Com.*, vol. COM-28, pp. 1343-1354, Aug. 1980.
- [10] G. S. Gill and S. C. Gupta, "First-order discrete phase-locked loop with applications to demodulation of angle-modulated carrier," *IEEE Trans. on Comm. Tech.*, vol. COM-20, no.7, pp.454-462, June 1972.
- [11] P. Z. Peebles, Jr., *Probability, Random Variables, and Random Signal Principles*, McGraw-Hill, New York, 1993.
- [12] J. Proakis and M. Salehi, *Communications Systems Engineering*, Prentice-Hall, 2002.

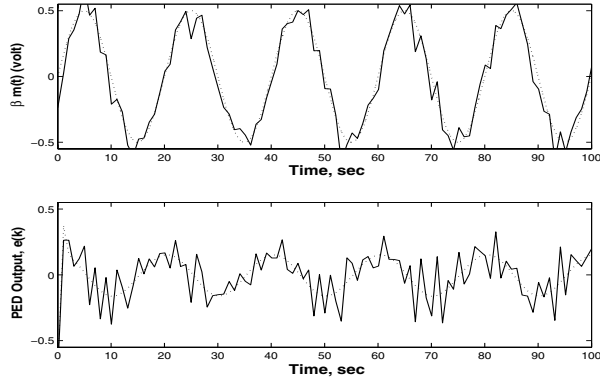


Fig. 7. TDTL demodulation for noisy tone PM signal with $A_m = 1$, $A_o = 1$, $f_o = 1$ Hz, $f_m = 0.05$ Hz, $\gamma_o = -0.5$ rad, $\psi_o = \pi/2$, and $\beta = 0.5$ and received SNR = 20 dB. Dotted curves are for the noiseless case.

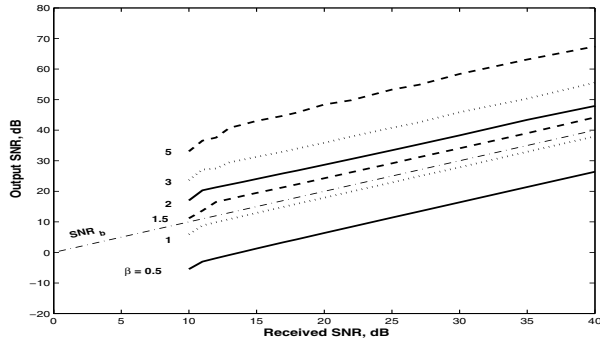


Fig. 8. Performance of TDTL for noisy tone PM demodulation with $A_m = 1$, $A_o = 1$, $f_o = 1$ Hz, $f_m = 0.05$ Hz, $\gamma_o = 0$, $\psi_o = \pi/2$, and different values for β . The dotted line labelled as SNR_b is for baseband transmission.

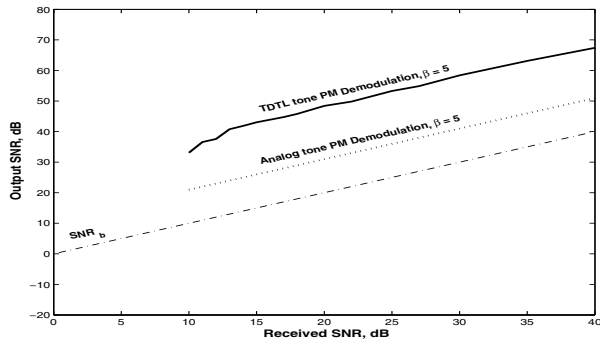


Fig. 9. Performance of TDTL demodulation versus analog demodulation for noisy tone PM signal with $A_m = 1$, $A_o = 1$, $f_o = 1$ Hz, $f_m = 0.05$ Hz, $\gamma_o = 0$, $\psi_o = \pi/2$, and $\beta = 5$. The dotted line labelled as SNR_b is for baseband transmission.